LAU
الجامعة اللبنانية الأميركية
Lebanese American University

LAU
School of Engineering

Department of Electrical
and Computer Engineering

ELE 201: Electrical Circuits I
Exam 1 – Spring 2014

Duration: **1 hour 20 minutes**
Start Time: 9:30 am

Date: 20/03/2014
Dr. Dani TANNIR

Name: SOLUTION **ID#:** _____

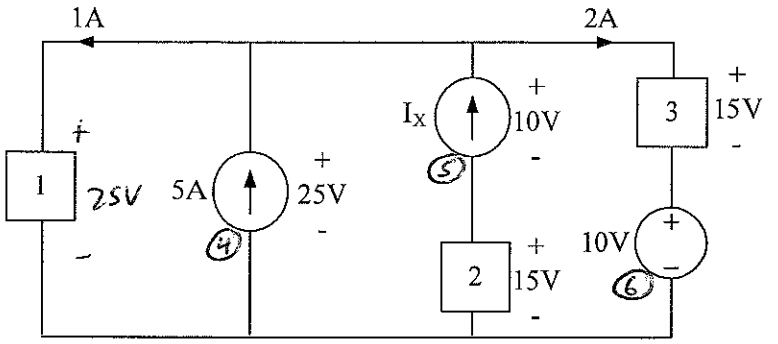
INSTRUCTIONS:

- Answer each of the following questions in the space provided.
- This is a closed-book exam.
- If something is not clear, state your assumptions.
- Programmable calculators are not allowed.
- The number of marks for each question is specified next to it.
- The total number of marks is 100.

1	2	3	4	Total
/20	/30	/25	/25	100

Question 1 (20 marks)

a. Find I_x in the network shown below **using the conservation of power principle** (power balance).
b. Specify which elements are supplying and which are receiving power.
c. Calculate how much charge enters element '3' over the interval 0 to 20s.
d. Calculate the total energy delivered or supplied by element '3' over the interval 0 to 20s.



a) $P_1 = 25W$

$P_2 = -15I_x$

$P_3 = 30W$

$P_4 = -125W$

$P_5 = -10I_x$

$P_6 = 20W$

$75W - 125W = -50W$

$25 + 30 + 20 - 125 - 25I_x = 0$

$25I_x = -50$

$I_x = -2A$

b) Receiving: ①, ②, ③, ⑥, ⑤

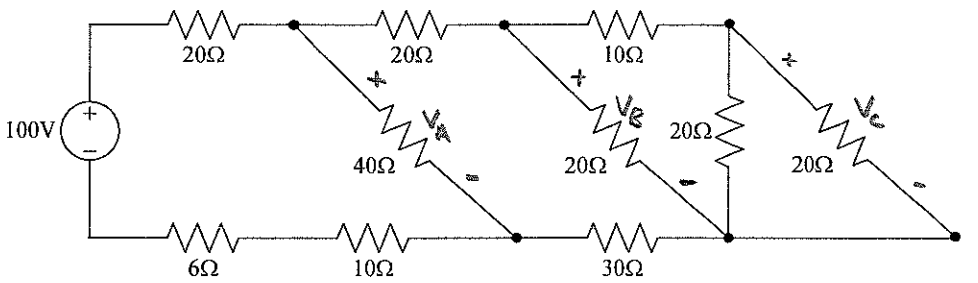
Supplying: ④

c) $q = \int_0^{20} \underbrace{2}_{(i)}\, dt = (2)(20) = \boxed{40C}$

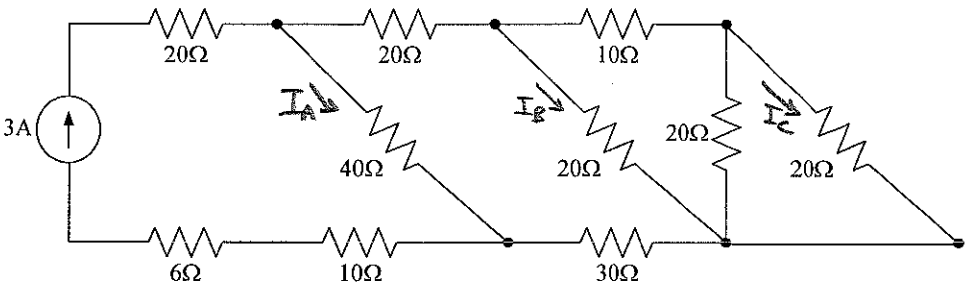
d) $w = \int_0^{20} \underbrace{30}_{(vxi)}\, dt = (30)(20) = \boxed{600J}$

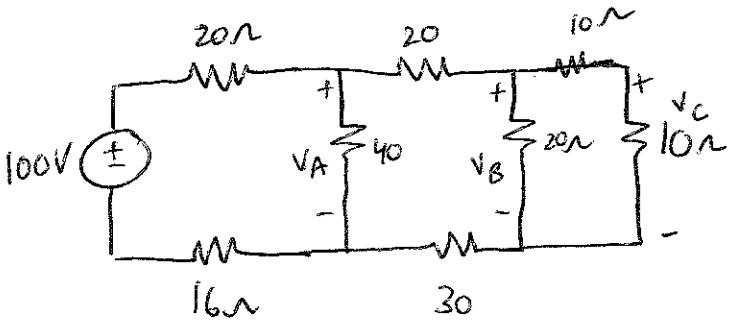
Question 2 (30 marks)

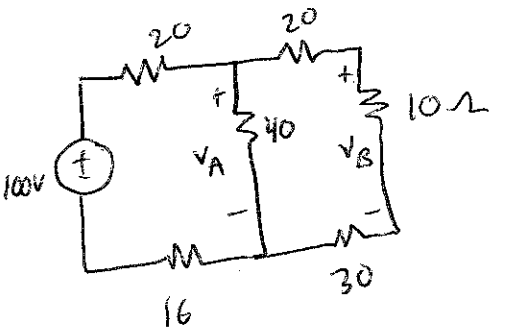
a. Use **voltage division** and series/parallel equivalent circuits to determine the voltages V_A, V_B and V_C as labelled.

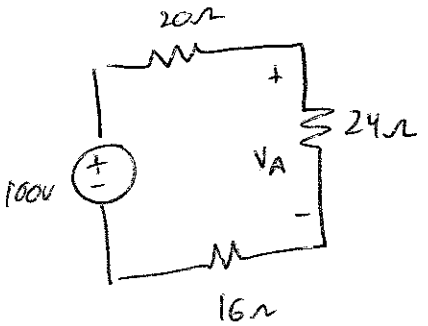


b. Use **current division** and series/parallel equivalent circuits to determine the currents I_A, I_B and I_C as labelled.









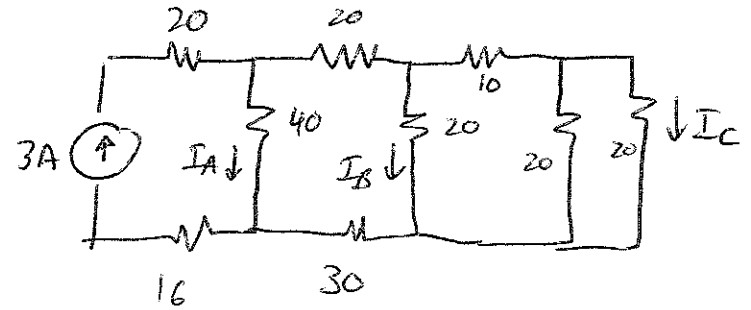
$$V_A = \frac{24}{20+16+24}\,100$$

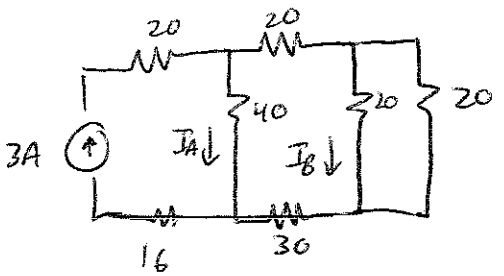
$$= 40V$$

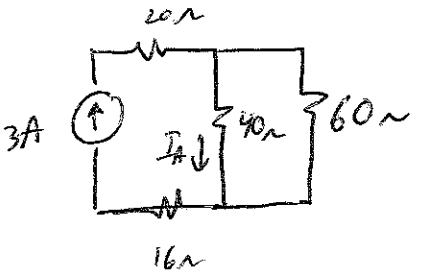
$$V_B = \frac{10}{60}\,V_A = \frac{40}{6}\,V \approx 6.67V$$

$$V_C = \frac{10}{10+10}\,V_B = \frac{40}{12}\,V = 3.33V$$

b)







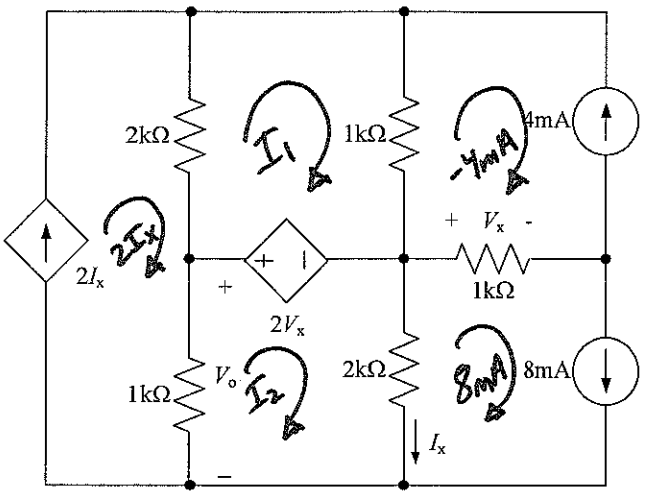
$$I_A = \frac{60}{40+60}(3A) = 1.8A$$

$$I_B = (3 - I_A)\frac{20}{20+20} = 0.6A$$

$$I_C = \underbrace{0.6}_{\text{same as } I_B}\ \frac{20}{20+20} = 0.3A$$

Question 3 (25 marks)

Use mesh analysis to determine the value of V_o in the following circuit



Mesh I_1

$$2k(I_1 - 2I_x) + 1k(I_1 - (-4mA)) - 2V_x = 0$$

Mesh I_2

$$2V_x + 2k(I_2 - 8m) + 1k(I_2 - 2I_x) = 0$$

$$I_x = I_2 - 8m$$

$$V_x = [8m - (-4m)] \Omega 1k = 12V$$

$$\Rightarrow 24 + 2kI_2 - 16 + 1kI_2 - 2k(I_2 - 8m) = 0$$

$$8 + 3kI_2 - 2kI_2 + 16 = 0$$

$$1kI_2 = -24$$

$$I_2 = -24mA$$

$$\Rightarrow I_x = -24m - 8m = -32mA$$

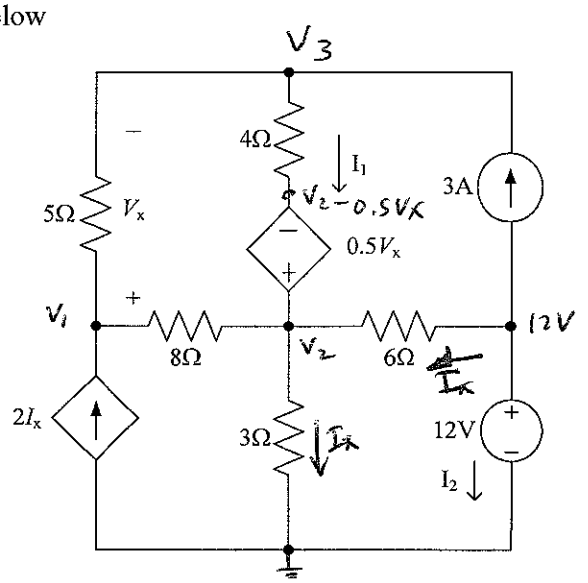
$$V_o = 1k(2I_x - I_2)$$

$$V_o = 1k(-64m + 24m)$$

$$= -40V$$

Question 4 (25 marks)

Consider the circuit shown below



$3 + I_x + I_2 = 0$

$I_2 = -3 - I_x$

Use Node Analysis to determine the values of I_1 and I_2 as labelled.

$V_x = V_1 - V_3$

$I_x = \frac{V_2}{3}$

Node equation @ V_1

$$-2\left(\frac{V_2}{3}\right) + \frac{V_1 - V_2}{8} + \frac{V_1 - V_3}{5} = 0$$

$$-2I_x + \frac{V_1 - V_2}{8} + \frac{V_1 - V_3}{5} = 0$$

Node equation @ supernode

$$\frac{V_2}{3} + \frac{V_2 - V_1}{8} + \frac{V_2 - 12}{6} + \frac{V_2 - 0.5(V_1 - V_3)}{4} - V_3$$

$$\frac{V_2}{3} + \frac{V_2 - V_1}{8} + \frac{V_2 - 12}{6} + \frac{V_2 - 0.5V_x}{4} - V_3 = 0$$

Node equation @ V_3

$$\frac{V_3 - V_1}{5} + \frac{V_3 - (V_2 - 0.5(V_1 - V_3))}{4} - 3 = 0$$

$$\frac{V_3 - V_1}{5} + \frac{V_3 - (V_2 - 0.5V_x)}{4} - 3 = 0$$

$$V_1\left(\frac{1}{8} + \frac{1}{5}\right) + V_2\left(-\frac{2}{3} - \frac{1}{8}\right) + V_3\left(-\frac{1}{5}\right) = 0$$

$$V_1\left(-\frac{1}{8} - \frac{0.5}{4}\right) + V_2\left(\frac{1}{3} + \frac{1}{8} + \frac{1}{6} + \frac{1}{4}\right) + V_3\left(\frac{0.5}{4}\right) = \frac{12}{6} = 2$$

$$V_1\left(-\frac{1}{5} + \frac{0.5}{4}\right) + V_2\left(-\frac{1}{4}\right) + V_3\left(\frac{1}{5} + \frac{1}{4} - \frac{0.5}{4}\right) = 3$$

$$0.325V_1 + (-0.7917)V_2 - 0.2V_3 = 0 \quad (1)$$

$$-0.25V_1 + 0.875V_2 - 0.125V_3 = 2 \quad (2)$$

$$-0.075V_1 - 0.25V_2 + 0.325V_3 = 3 \quad (3)$$

$$V_1 = -95.087\,V$$

$$V_2 = -30\,V$$

$$V_3 = -35.785\,V$$

$$\Rightarrow V_x = -59.3\,V$$

$$I_1 = \frac{V_3 - (V_2 - 0.5V_x)}{4} = \boxed{-8.86\,A}$$

$$I_2 = -3 - \frac{(12+30)}{6}$$

$$= \boxed{-10A}$$